

Introduction

During recent years *Quantum Information Theory* (QIT) has become a very active field of research at the intersection of Mathematics, Physics and Computer Science. It deals with the generalization of information and communication theory from classical to quantum systems. One of the most fundamental originating concepts is the notion of *entanglement*, which describes a non-classical correlation between different subsystems.

Entanglement I - Qubits

Qubits are two-level quantum systems, which can be described as positive, normalized, trace-class operators over complex-valued Hilbertspaces. Using Dirac's bra-ket notation, we can for example write

$$\rho = (|0\rangle\langle 0| + |1\rangle\langle 1|) / 2, \quad (1)$$

where ρ is the so called *density matrix* and $|0\rangle, |1\rangle \in \mathbb{H}$ are the two orthogonal basis vectors. More complex systems consisting of several qubits (or higher dimensional subsystems) are modeled on tensor product spaces $\mathbb{H} = \mathbb{H}_1 \otimes \dots \otimes \mathbb{H}_n$. A standard example of a (maximally) entangled two qubit system is

$$\rho = |\Phi^+\rangle\langle\Phi^+| = \frac{1}{2}(|0\rangle|0\rangle + |1\rangle|1\rangle)(\langle 0|\langle 0| + \langle 1|\langle 1|). \quad (2)$$

The two qubits are perfectly correlated, even though their local states are completely mixed (namely equal to ρ in (1)). A measurement on one subsystem will immediately influence the other one, regardless of the distance between both.

Entanglement Measures

In order to quantify the amount of entanglement in a given density matrix ρ so called *entanglement measures* like the *entropy of entanglement*

$$E_{EoE}(\rho_{AB}) = S(\underbrace{\text{tr}_B \rho}_{\equiv \rho_A}) = -\text{tr}_A[\log(\rho_A)\rho_A] \quad (3)$$

for *pure states* ($\Leftrightarrow \text{tr}[\rho^2] = 1$) or the *Distillable entanglement*

$$E_D(\rho) \equiv \sup_{m(n) \in \mathbb{N}} \left\{ \frac{m}{n} : \lim_{n \rightarrow \infty} \inf_{\text{LOCC } \Lambda} \|\Lambda(\rho^{\otimes n}) - \Phi(2^m)\|_1 = 0 \right\} \quad (4)$$

have been introduced. They are often connected to an operational task and many are very hard or impossible to evaluate in general. As it turns out, entanglement can be view as a resource, which can be transferred between systems.

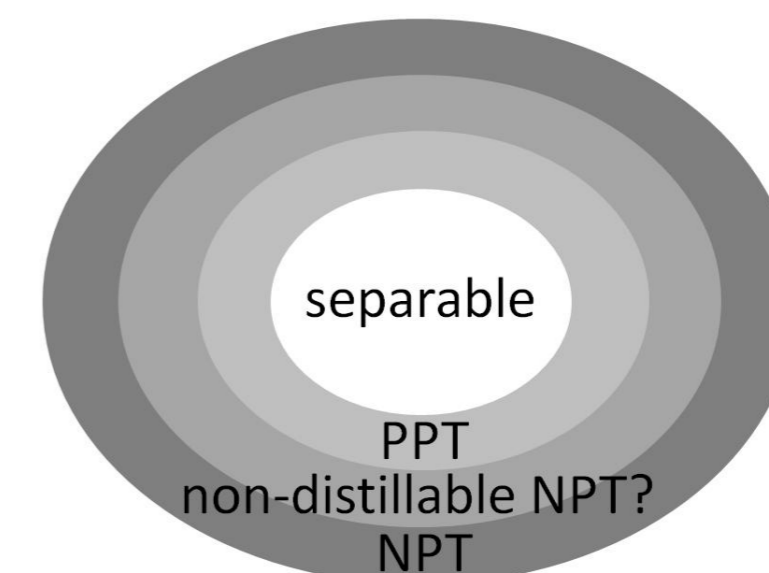
References

- [1] R. Wißmann. Entanglement, Distillation and Quantum Repeaters: From Qubits to Gaussian States. Bachelor thesis, 2008.
- [2] W. Dür, H.-J. Briegel, J. I. Cirac, and P. Zoller. Quantum repeaters based on entanglement purification. *Phys. Rev. A*, 59:169, 1999.
- [3] M. M. Wolf, J. Eisert, G. Giedke, J. I. Cirac, M. Lewenstein, M. B. Plenio, and R. F. Werner. Gaussian states and operations (textbook in preparation).

Distillation

Often one has a large supply of low fidelity states but needs highly entangled, nearly pure ones. So called *distillation protocols* [2] can be used to extract these. The overall structure of the set of *distillable* states is shown in picture 1, where PPT (NPT) refers to (*non*-)positive partial transpose as defined by

$$\rho_{AB}^{T_B} \geq (\not\geq) 0. \quad (5)$$



Picture 1. Comparison of different important sets of bipartite states. (after Horodecki et al., q-ph/0702225)

Entanglement II - Gaussian Setting

If instead of discrete levels one considers continuous variables the underlying Hilbert space becomes infinite dimensional and the mathematical formalism becomes very involved. Fortunately, the most important and experimentally accessible set of states is given by so called *Gaussian states*

$$\rho = \int_{\mathbb{R}^{2N}} e^{-\xi^T \gamma \xi / 4 + i \xi^T d'} W_{-\xi} d\xi, \quad (6)$$

where $\gamma \in \mathbb{R}^{2N \times 2N}$ (covariance matrix), $d \in \mathbb{R}^{2N}$ (displacement vector), $\gamma' = \sigma^T \gamma \sigma$, $d' = \sigma d$ and $\gamma + i\sigma \geq 0$. In eq. (6) we used *Weyl operators* W_ξ as defined by

$$W_\xi \equiv e^{i \xi^T \sigma R} \quad \forall \xi \in \mathbb{R}^{2N} \quad (7)$$

and the *canonical commutation relations* $[R_k, R_l] = i\sigma_{kl} \mathbb{1}$. Using this *Weyl formalism* it is possible to derive analogous notions of entanglement theory, entanglement measures and distillation. [3]

Quantum Repeaters

In order to create long-range, highly entangled states, distillation and *quantum teleportation* have been combined into *Quantum Repeaters* (QR), which use iterative techniques. [2] QRs have originally been derived for the finite-dimensional case and it is not yet clear, if and how they can be adopted into the Gaussian setting. While some important intermediary results have been found, other parts of this question still remain open (see e.g. [3]).