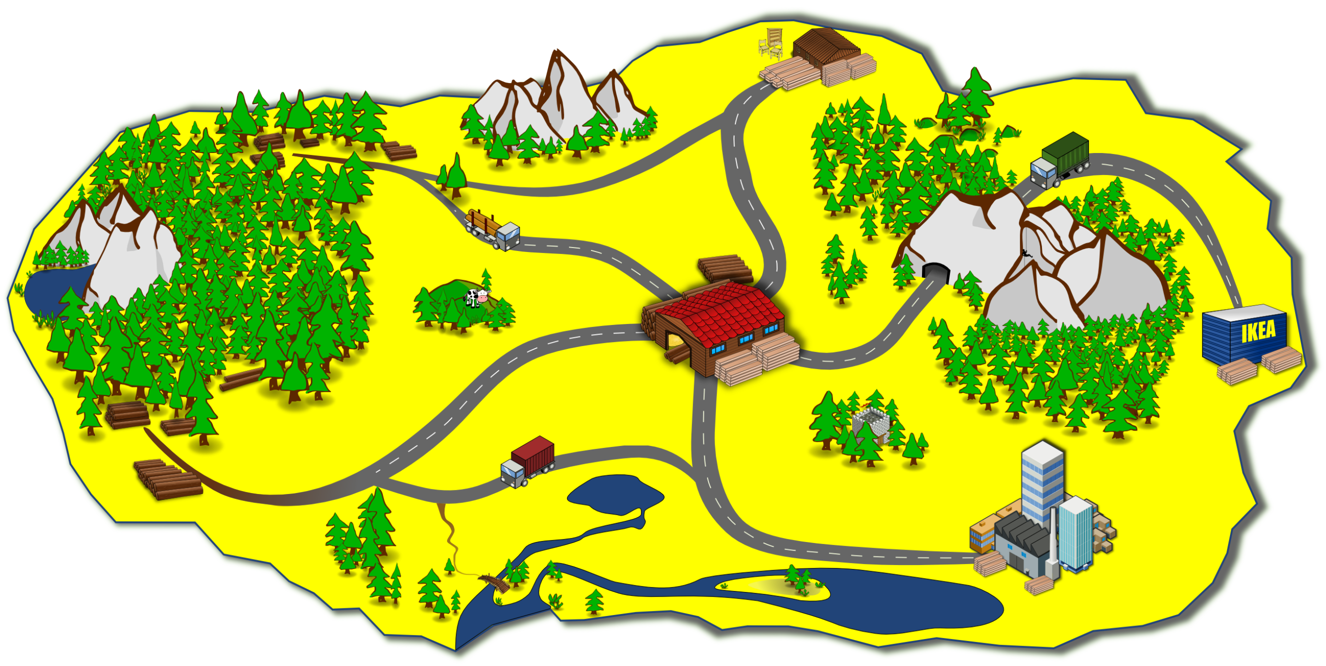
Vehicle Routing with Flexible Load Carriers

Paul Stursberg
Center for Mathematical Sciences, Technische Universität München
Chair of Applied Geometry and Discrete Mathematics

Abstract

In the logging industry and in related contexts, a routing problem arises that considers a number of vehicles which use containers to fulfil a set of transportation tasks. To begin a transportation task, a container needs to be present at the task's place of origin, which makes it necessary to develop a strategy for the distribution of containers to these places. The necessity to keep track of the containers presents a major new difficulty compared to other Vehicle Routing Problems which necessitates the usage of new solution approaches and in turn opens new connexions to other problems such as the well-known Airline Crew Scheduling.

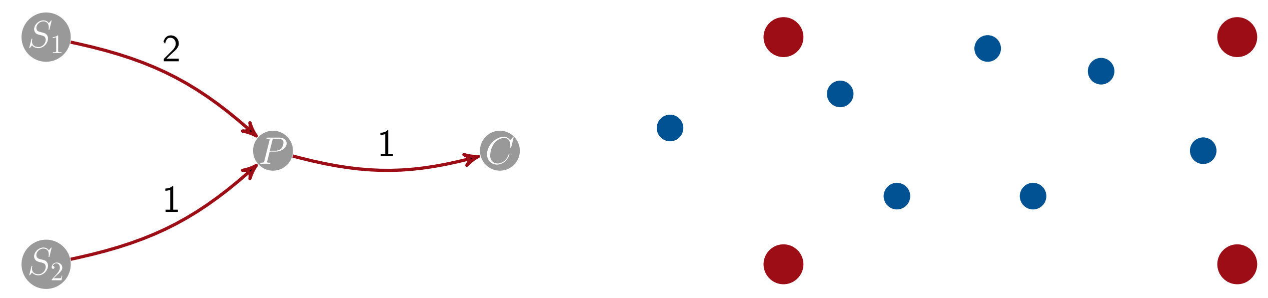
Setting



Timber from the forests on the left must be transported to the central sawmill. From there, sawn wood is carried on to processing sites on the right.
In contrast to the classical solution, where different transporters are used for timber and sawn wood respectively, we investigate a new approach which uses multifunctional foldable containers, that may be used for either commodity. In addition, while empty, several (up to 4) containers can be transported at the same time.

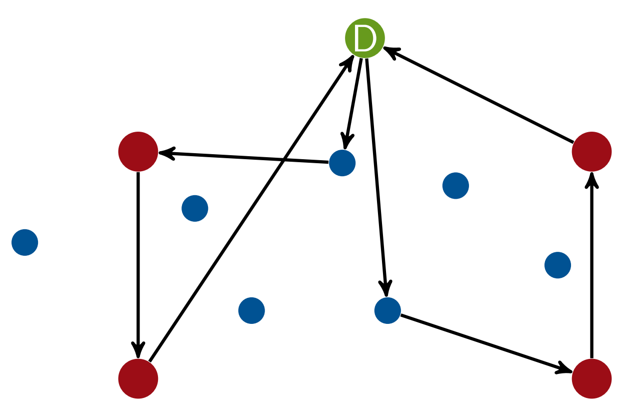
Vehicle Flow Model

To model the problem in the fashion of a Vehicle Routing Problem with Time Windows (see [2]), we employ the concept of the Task Graph:



original Graph Task Graph

- use transportation tasks as vertices
- add vertices representing empty container trips
- route vehicles on the resulting graph (ordinary Vehicle Routing Problem)
- add special constraints to regulate container usage



To track starting times and to regulate container usage, the following types of typically very weak inequalities appear:

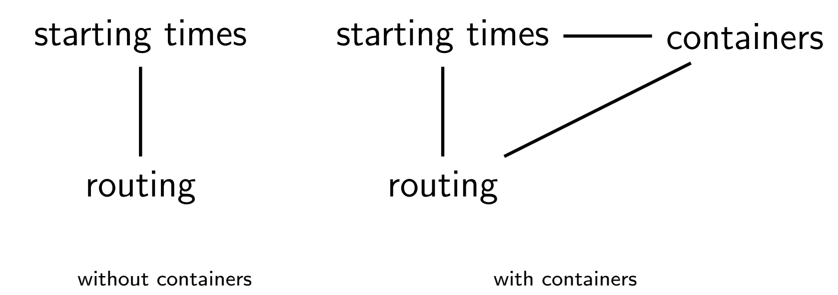
$$t_i + T_{ij} \leq t_j + M \cdot (1 - x_{ij})$$
$$t_i + d_i \leq t_j + M \cdot (1 - c_{ij})$$

We were able to achieve a stronger and faster LP-Relaxation by applying the following modifications:

- simplifying the model by relaxing a number of constraints
- employing a different time formulation due to [3]
- reformulating the container inequalities as a separate container flow

Decomposition

If we represent direct dependence between classes of variables as edges of a graph, the inclusion of containers into the problem makes this graph 2-connected.



This implies that the model cannot be decomposed in any useful way. Most successful solution strategies for the Vehicle Routing Problem with Time Windows, however, rely on decomposition and are therefore not applicable.

Min-Vehicle-Cut

Vehicles often have to respect certain limitations (e.g. due to labour regulations) regarding the maximal duration of their trips. However, the constraints that enforce these limitations increase the problem size significantly.

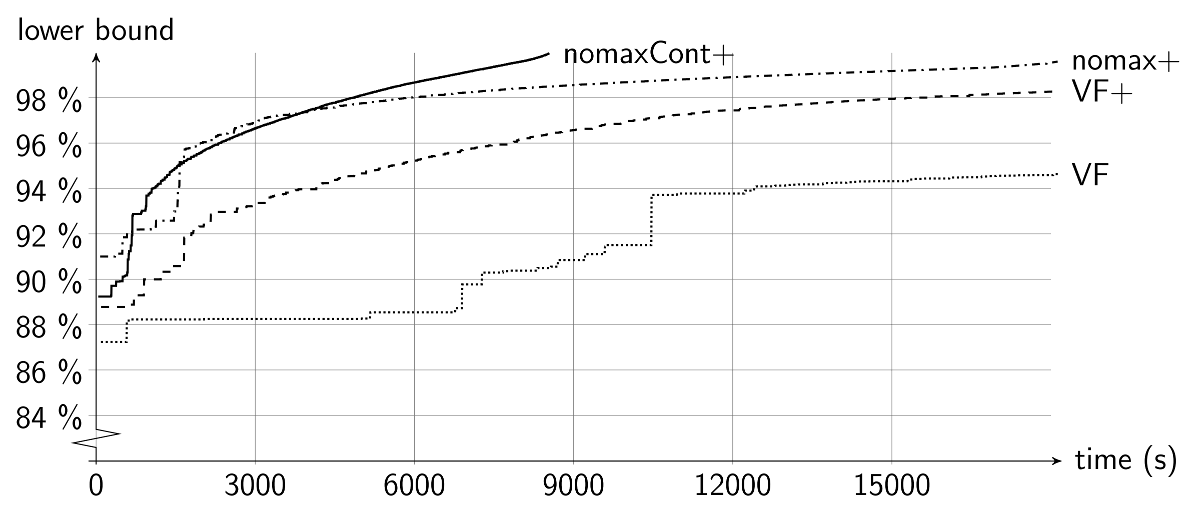
- replace these constraints by an estimate on the number of needed vehicles:

$$\sum_{k \in D_S} \sum_{j \in N} x_{kj} \geq \left\lceil \frac{z_{LP} - \sum_{(i,j)\in A} \bar{w}_{ij}}{T} \right\rceil$$

where z_{LP} be the value of the LP-Relaxation and w the accumulated waiting time of all vehicles in the corresponding solution. Let T be the maximal driving time that any vehicle can spend on a single trip.

Computational Results

Using the strategies referred to on the left, computation time and quality of lower bounds increases significantly: The Branch-and-Bound-algorithm terminates with the optimal solution at a time, when the bound generated by the original model has only just surpassed 90 % of the optimal solution.
The plot shows lower bounds arising in a standard branch-and-bound-approach using different LP-Models for an instance with 12 full container tasks: VF (Vehicle Flow), nomax (simplification), Cont (new container flow), + (cut based on lower bound on number of vehicles)



References

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[2] J. Desrosiers, F. Soumis, and M. Desrochers, and. "Routing with time windows by column generation". In: *Networks* 14.4 (1984), pp. 545–565.

[3] A. Langevin, M. Desrochers, J. Desrosiers, S. Gélinas, and F. Soumis, and. "A two-commodity flow formulation for the traveling salesman and the makespan problems with time windows". In: *Networks* 23.7 (1993), pp. 631–640.

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