Optimal Scheduling and Routing in Extramural Health Care

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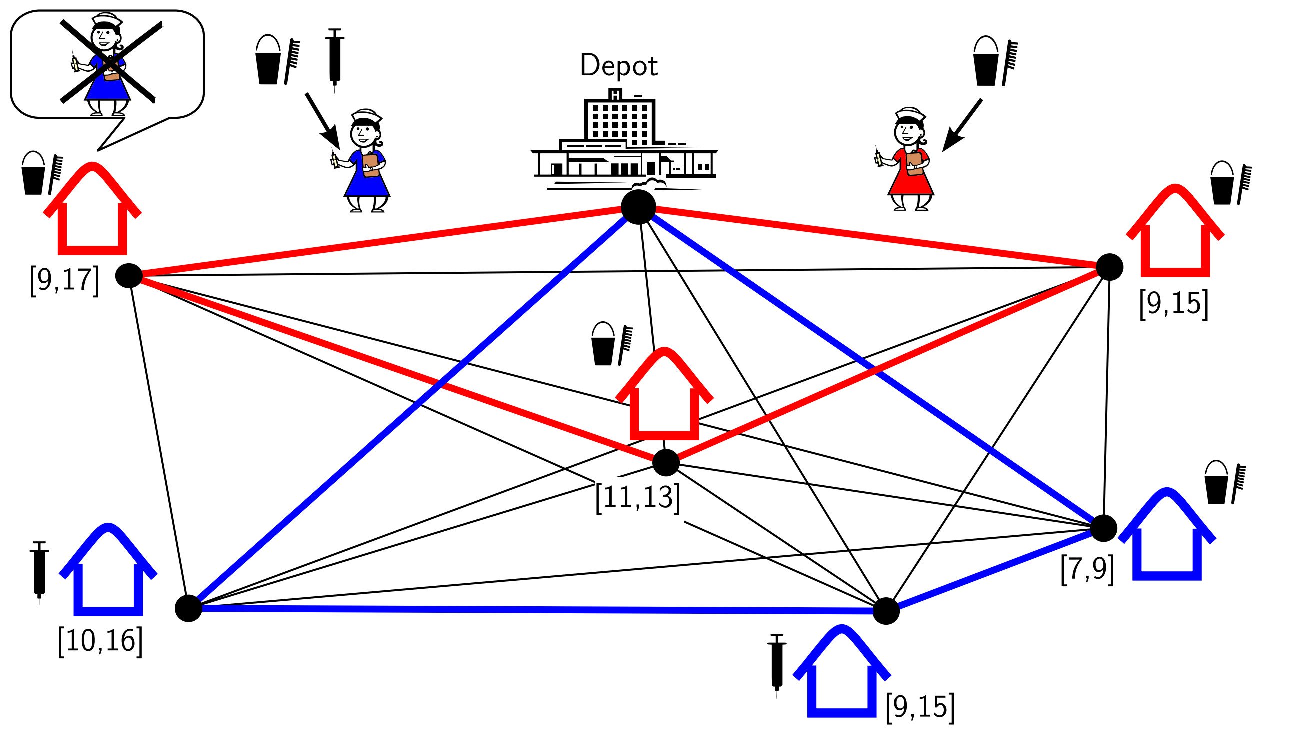
Abstract

Due to demographic change, the demand for extramural health care will rise. Therefore, efficient scheduling and routing of nurses is needed in order to minimize driving and waiting times. Different ILP formulations can be used to model this extension of the *Vehicle Routing Problem with Time Windows*. Lower bounds on the optimal value are obtained by a Lagrangian relaxation using a subgradient method. A set partitioning approach is solved by column generation to achieve feasible solutions. In both cases, the arising subproblems are solved via a modification of the algorithm of Ioachim et al. (1998).

Problem Description



Given a set $\mathcal{J}$ of jobs and a set $\mathcal{V}$ of different shifts of the nurses, a route configuration covering each job has to be found. In addition the following requirements have to be met:

- hard time windows
- mandatory breaks
- assignment restrictions originating from different qualification levels, language skills or personal dislike
- different contract types of the nurses resulting in different working times

Among all feasible solutions, one with minimal driving and waiting times as well as a small dissatisfaction level is searched. The dissatisfaction level is measured using different factors such as violated soft time windows of customers and nurses, overtime, overqualification of nurses or unsatisfied preferences of customers.

Different ILP Models

The problem described above can be modelled as an ILP. A simplified description is given below using the following variables:

$$s_{ik} : \text{Starting time of job } i \text{ in tour } k$$

$$x_{ijk} : \begin{cases} 1 & \text{Job } i \text{ is visited by tour } k \text{ right after job } j \\ 0 & \text{otherwise} \end{cases}$$

$$START_k, END_k : \text{Starting and ending time of tour } k$$

[3-Index]

$$\min \sum_{k \in \mathcal{V}} (END_k - START_k) - \sum_{j \in \mathcal{J}} d_j$$

s.t.

$$\sum_{j \in \mathcal{J}_0, k \in \mathcal{V}} x_{ijk} = 1 \qquad \forall i \in \mathcal{J} \quad (1)$$

$$\sum_{h \in \mathcal{J}_0} x_{hik} = \sum_{j \in \mathcal{J}_0} x_{ijk} \qquad \forall i \in \mathcal{J}, \forall k \in \mathcal{V} \quad (2)$$

$$\sum_{j \in \mathcal{J}} x_{0jk} \leq 1 \qquad \forall k \in \mathcal{V} \quad (3)$$

$$a_i \sum_{j \in \mathcal{J}_0} x_{ijk} \leq s_{ik} \leq b_i \sum_{j \in \mathcal{J}_0} x_{ijk} \qquad \forall i \in \mathcal{J}, \forall k \in \mathcal{V} \quad (4)$$

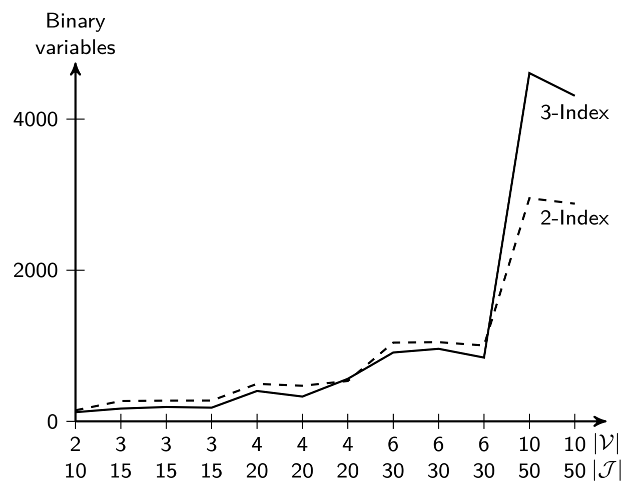
$$s_{ik} + (d_i + t_{ij})\, x_{ijk} \leq s_{jk} + M\,(1 - x_{ijk}) \qquad \forall i, j \in \mathcal{J}, \forall k \in \mathcal{V} \quad (5)$$

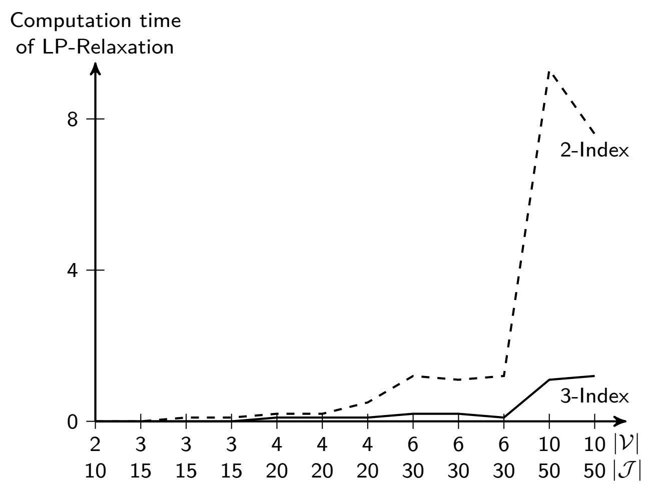
$$s_{ik} + (d_i + t_{i0})\, x_{i0k} \leq END_k + M\,(1 - x_{i0k}) \qquad \forall i \in \mathcal{J}, \forall k \in \mathcal{V} \quad (6)$$

$$START_k + t_{0j} x_{0jk} \leq s_{jk} + M\,(1 - x_{0jk}) \qquad \forall j \in \mathcal{J}, \forall k \in \mathcal{V} \quad (7)$$

$$x_{ijk} \in \{0, 1\} \qquad \forall i, j \in \mathcal{J}_0, \forall k \in \mathcal{V} \quad (8)$$

An equivalent description using only two-indexed variables yields a model with less binary variables. Unfortunately, an increase in the computation time of the linear relaxation does not yet allow any improvement in the computation of optimal solutions.





Lagrangian Relaxation

By adding constraints that link different shifts ((1) in the presented model) to the objective function, one obtains a formulation that decomposes into independent subproblems, one for each shift:

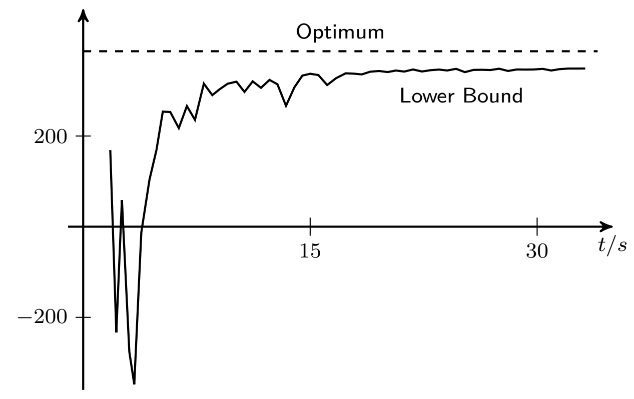
[Lag]

$$\min \sum_{k \in \mathcal{V}} (END_k - START_k) - \sum_{j \in \mathcal{J}} d_j + \sum_{i \in \mathcal{J}} \lambda_i \left(1 - \sum_{j \in \mathcal{J}_0, k \in \mathcal{V}} x_{ijk}\right)$$

s.t. $(2) - (8)$

The resulting subproblem is a special case of the $\mathbb{ESPPTWTC}$, whose relaxation $\mathbb{SPPTWTC}$ can be solved in pseudo-polynomial time using the algorithm of Ioachim et al. (1998). The gap between $\mathbb{ESPPTWTC}$ and $\mathbb{SPPTWTC}$ can be tightened by eliminating 2-cycles (circles of the form $\ldots - i - j - i - \ldots$) from the solution space following Houck (1978). As not all of the original constraints and factors of the objective function can be extended to the $\mathbb{SPPTWTC}$, the algorithm often just returns a lower bound of the optimal solution.

By adjusting the Lagrangian multipliers via a subgradient method, a maximal lower bound is searched. Computational experiments on 10 instances of different sizes show that the relaxation is able to provide lower bounds approximating the optimal solution at an average of 4.1%. An equivalent approach using *Variable Splitting* yields similar results with an increased computation time.



Set Partitioning via Column Generation

Using the set $\mathcal{S}$ of all possible tours of a shift with corresponding cost c_s and shift k_s, a set partitioning formulation with an exponential number of variables can be obtained. For the simplified problem the model is the following:

[SC]

$$\min \sum_{s \in \mathcal{S}} c_s y_s$$

s.t.

$$\sum_{s \in \mathcal{S}: i \in s} y_s = 1 \qquad \forall i \in \mathcal{J}$$

$$\sum_{s \in \mathcal{S}: k_s = k} y_s \leq 1 \qquad \forall k \in \mathcal{V}$$

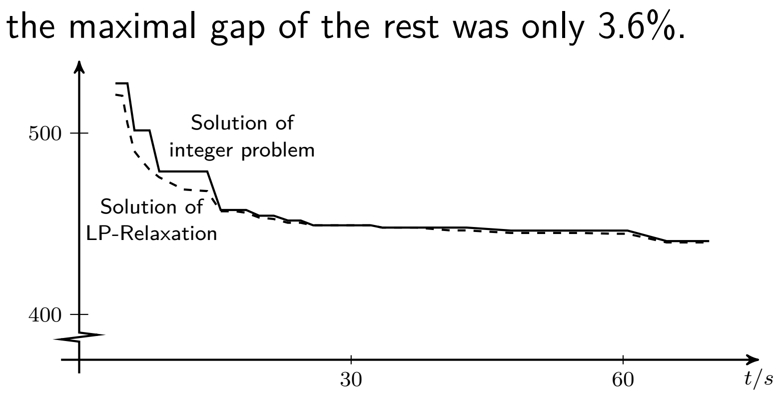
$$y_s \in \{0, 1\} \qquad \forall s \in \mathcal{S}$$

Due to the huge number of variables, the linear relaxation of this model is solved by column generation. Duality theory yields the following criteria for new columns with negative reduced cost:

$$c_s - \sum_{i \in \mathcal{J}: i \in s} \pi_i + \mu_{k_s} < 0$$

with $\pi_i \in \mathbb{R},\ i \in \mathcal{J}$ and $\mu_k \geq 0,\ k \in \mathcal{V}$ dual variables of **[SC]**.

The result of the model with binary variables gives an upper bound of the optimal solution. In computational experiments, 7 out of 10 instances could be solved to optimality, while the maximal gap of the rest was only 3.6%.



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