

Montag, 1.12.2008 Fink Holger (TU München)

Functions of bounded p -variation

Let h be a right continuous function on a compact interval K . Using the Banach-Steinhaus theorem one can show that if for all continuous function g the Riemann-Stieltjes (RS) sums

$$\sum_{i=1}^n g(y_i)[h(x_{i+1}) - h(x_i)]$$

converge for all subdivisions $(x_i)_{i=1, \dots, n+1}$ and intermediate subdivisions $(y_i)_{i=1, \dots, n}$ of K , then h is of bounded variation.

Is it possible to restrict the space of integrands and get convergence of the RS sums for an integrator which is not of bounded variation?

An answer can be given with the following concept: A function h on K is called of bounded p -variation, $p > 0$, if and only if

$$\sup_{\kappa} \sum_{i=1}^n |h(x_{i+1}) - h(x_i)|^p < \infty$$

where the supremum is taken over all subdivisions κ of K . In this talk we will shortly introduce that concept and prove the convergence of the RS sums for suitable integrands. Applications to stochastic calculus and financial mathematics are given.